Radiation Physics and Chemistry 188 (2021) 109651

Contents lists available at ScienceDirect

Radiation Physics and Chemistry

journal homepage: www.elsevier.com/locate/radphyschem

Calculation of exposure and absorbed dose at irradiation samples by electron bremsstrahlung

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ARTICLE INFO

Keywords:
Exposure dose
Absorbed dose
Electron bremsstrahlung
Thin target
Thick target

ABSTRACT

Current work describes a technique devoted to calculating the exposure and absorbed doses at irradiation of samples by electron bremsstrahlung. The developed approach is based on using several well-known semi-empirical formulas that describe the bremsstrahlung of electrons and tabular values of the mass absorption coefficient of photon energy. The calculation algorithm is implemented in the Wolfram Mathematica and is available for free use.

1. Introduction

Ionizing radiation has many practical applications – from creating new materials (Wu et al., 2018) to the destruction of harmful biological organisms in foodstuff (Miller, 2005). Research in this area, being at the intersection of various fields of science, is mainly carried out using isotopic γ-sources and electron accelerators. The bremsstrahlung generated by accelerators is one of the most intense sources of ionizing radiation. Unlike isotopic γ-sources, the bremsstrahlung spectrum is a continuous distribution of a rather complex shape, the photon energy that extends to electrons' kinetic energy. In addition, bremsstrahlung has a significant angular anisotropy, the value of which is determined by the thickness of the target and the electron energy. Therefore, calculating the values of the exposure and absorbed doses when the samples are irradiated by bremsstrahlung is a nontrivial problem. This paper describes a relatively simple algorithm that calculates the exposure and absorbed doses when the samples are irradiated by electron bremsstrahlung. The calculation is based on the use of a number of well-known semi-empirical formulas that describe the bremsstrahlung and tabular values of the mass absorption coefficient of photon energy. The algorithm is implemented in Wolfram Mathematica and available for free use.[1]

2. The energy spectrum of electron bremsstrahlung

An accurate description of electron bremsstrahlung (*further bremsstrahlung*) can be obtained within the framework of quantum electrodynamics (Heitler, 1954). Bethe and Heitler first obtained an analytical expression for the double differential cross section of the bremsstrahlung of relativistic electrons based on a number of assumptions (Bethe and Heitler, 1934). Subsequently, with the development of the formalism of quantum electrodynamics, more accurate results were obtained, a modern review of which is given in (Mangiarotti and Martins, 2017).

In practice, when calculating the parameters of bremsstrahlung from a thin target,[2] the double differential cross-section obtained by Schiff (1951) is usually used:

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[1] URL: http://dxdydz.xyz/bremsstrahlung/.

[2] *Thin* means a target whose thickness t is much less than the radiation length for the corresponding target material, i.e. $t \ll 1$.

https://doi.org/10.1016/j.radphyschem.2021.109651
Received 20 March 2021; Received in revised form 9 June 2021; Accepted 15 June 2021
Available online 19 June 2021

$$\frac{d\sigma(k,x)}{dk\,dx}=\frac{4Z^2}{137}\left(\frac{e^2}{m_ec^2}\right)^2\frac{x}{k}\left\{\frac{16x^2E}{(x^2+1)^4E_0}-\frac{(E_0+E)^2}{(x^2+1)^2E_0^2}+\left[\frac{E_0^2+E^2}{(x^2+1)^2E_0^2}-\frac{4x^2E}{(x^2+1)^4E_0}\right]\ln M\left(x\right)\right\},\qquad(1)$$
$$\frac{1}{M(x)}=\left(\frac{k\,m_ec^2}{2E_0E}\right)^2+\left(\frac{Z^{1/3}}{C(x^2+1)}\right)^2,\ x=\frac{E_0\theta}{m_ec^2},\ C=111,$$

where E_0-is the total energy of the incident electron, E -is the total energy of the electron after scattering in the field of the atomic nucleus, $k=E_0-E$-is the energy of the emitted photon, θ -is the angle between the momentum vectors of the incident electron and the emitted photon, Z- is the atomic number of the target material.

Integration of formula (1) over all angles of emission of photons produces the spectral distribution of bremsstrahlung (Schiff's formula):

$$\frac{d\sigma(k)}{dk}=\frac{2Z^2}{137}\left(\frac{e^2}{m_ec^2}\right)^2\frac{1}{k}\left\{\left(\frac{E_0^2+E^2}{E_0^2}-\frac{2E}{3E_0}\right)\times\left(\ln M+1-\frac{2}{b}\arctan b\right)+\frac{E}{E_0}\left[\frac{2}{b^2}\ln(1+b^2)+\frac{4(2-b^2)}{3b^3}\arctan b-\frac{8}{3b^2}+\frac{2}{9}\right]\right\},$$
$$\frac{1}{M}=\left(\frac{k\,m_ec^2}{2E_0E}\right)^2+\left(\frac{Z^{1/3}}{C}\right)^2,\ b=\frac{2E_0E\,Z^{1/3}}{C\,k\,m_ec^2},\ C=111.\qquad(2)$$

Schiff's formula (2) represents the shape of the bremsstrahlung spectrum in the forward direction for infinitely thin targets. Therefore, this formula cannot be used in its current form to describe the bremsstrahlung spectra generated by electron accelerators with thick targets.[3] Later it was found that Schiff's formula with a coefficient $C=191$ better represents the real spectrum of bremsstrahlung in the forward direction from thick targets, where multiple passes and scattering of electrons make an additional contribution to the lower energy part of the bremsstrahlung curve (Katz and Cameron, 1951).

Schiff's formula (2) asymptotically tends to infinity at $k\to 0$. Therefore, from a practical point of view, when calculating the exposure and absorbed doses, it is much more convenient to work with the function of the intensity of bremsstrahlung normalized to unity:

$$I\left(k\right)=\left(\int_0^{E_0-m_ec^2}\frac{d\sigma(k)}{dk}k\,dk\right)^{-1}\times\frac{d\sigma(k)}{dk}k.\qquad(3)$$

The intensity function $I(k)$ is a probability density function that determines the contribution of photons energy of which lies between k and $k+dk$ to the total bremsstrahlung energy.

Fig. 1 illustrates the plots of the bremsstrahlung intensity function$I(k)$ for $E_0=5$MeV, $E_0=7\,MeV$, and a tantalum target ($Z=73$) constructed by formula (3).

3. The angular distribution of bremsstrahlung

In the original work of Schiff (1951), a double differential cross-section is given by (1), which describes the energy-angular distribution of bremsstrahlung. However, this formula is applicable only for thin targets, and in the case of thick targets, it gives considerable uncertainties in the description of the angular distribution since it does not take into account the processes of multiple scattering of electrons, absorption of photons in the target material, and other factors. Therefore, using semi-empirical formulas to calculate the angular distribution of bremsstrahlung from a thick target makes sense.

[3] *Thick* means a target whose thickness t is comparable to the radiation length for the corresponding target material, i.e. $t\sim1$.

In (Koch and Motz, 1959), the following semi-empirical formulas are given that describe the angular distribution of bremsstrahlung from a thick target:

$$R_{\alpha\gg E_0^{-1}}=-\frac{K\,E_0^2}{1760\pi}\mathrm{Ei}\left(\frac{-E_0^2\alpha^2}{1760t}\right),$$
$$R_{\alpha\le E_0^{-1}}=\frac{K\,E_0^2}{1760\pi}\left\{-\mathrm{Ei}\left(\frac{-E_0^2\alpha^2}{1760t}\right)+\mathrm{Ei}\left(\frac{-E_0^2\alpha^2}{7.15}\right)\right\},\qquad(4)$$

where α–the angle between the vectors of the momenta of the electron and the emitted photon (in radian units), $R(\alpha)$–fraction of the total incident electron kinetic energy that is radiated per steradian at the angle α, E_0–total energy in m_ec^2 units of the electron's rest energy, incident on the target, t–target thickness, in units of radiation length[4](Tsai, 1974) and $\mathrm{Ei}(z)=-\int_{-z}^{\infty}e^{-t}/t\,dt$ is an exponential integral. Radiation probability correction factor K is very little dependent on Z or E_0 and for heavy elements approximately equals to 0.7 in the energy range 5–10 MeV (Koch and Motz, 1959; Tsai, 1977; Koch, 1954). As noted in (Koch and Motz, 1959), the above formulas are in excellent agreement with the available experimental data.

Fig. 2 shows a graph (in polar coordinates) of the dependence of the bremsstrahlung power on the photon emission angle, constructed according to formula (4).

To obtain the value of efficiency for bremsstrahlung production η , we need to integrate a function $R(\alpha)$ over all emission angles:

$$\eta=\int_{4\pi}R(\alpha)d\Omega=2\pi\int_0^{\pi}R(\alpha)\sin\alpha d\alpha.\qquad(5)$$

For instance, for the tantalum target with a thickness of 1.2 mm, formula (5) produces $\eta\approx 13\,\%$ for $E_0=5\,MeV$ and $\eta\approx16\,\%$ when $E_0=7\,MeV$.

Photon fluence $\Phi(k)=dN/(dS\,dt\,dk)$ is defined as the number of photons dN energy of which lies between k and $k+dk$ crossing the elementary area dS over time dt. In the forward direction ($\alpha=0$) photon fluence can be easily calculated as follows:

$$dN=\frac{W\,R(0)\,d\Omega\,dt\,I(k)\,dk}{k},\ d\Omega=\frac{dS}{r^2}\Rightarrow\Phi(k)=\frac{dN}{dS\,dt\,dk}=\frac{W\,R(0)\,I(k)}{k\,r^2},\qquad(6)$$

where W–is the electron beam power (in Watt units).

Fig. 3 shows the graphs of bremsstrahlung photon fluence calculated by formula (6) for $dk=0.5$ MeV, $dt=1$ s, $dS=1$ cm^2, $r=1$ m, target material – tantalum, target thickness – 1.2 mm, electron beam current – 1 μA, initial total energy of electrons in an electron beam 5 and 7 MeV.

4. Calculation of exposure and absorbed doses

In the general case, the absorbed dose calculation should consider all types of interaction of photons with matter (photoelectric effect, Compton scattering, pair production, etc.). When describing the passage of monoenergetic photons through a matter, the mass attenuation coefficient is introduced:

$$\mu_m(k)=-\frac{1}{\rho\,N}\frac{dN}{dl},\qquad(7)$$

where N–is the number of photons with energy k incident typical to the material, dN– is the number of photons interacting with a layer of matter of thickness dl, ρ–is the density of matter.

[4] For instance, let the thickness of the tantalum target be 0.12 cm. Density of tantalum is 16.65 g × cm^{-3}. Radiation length for tantalum (Z = 73) is 6.8177 g × cm^{-2}. So thickness of the target, in radiation length units is 16.65 × 0.12/6.8177 = 0.29.

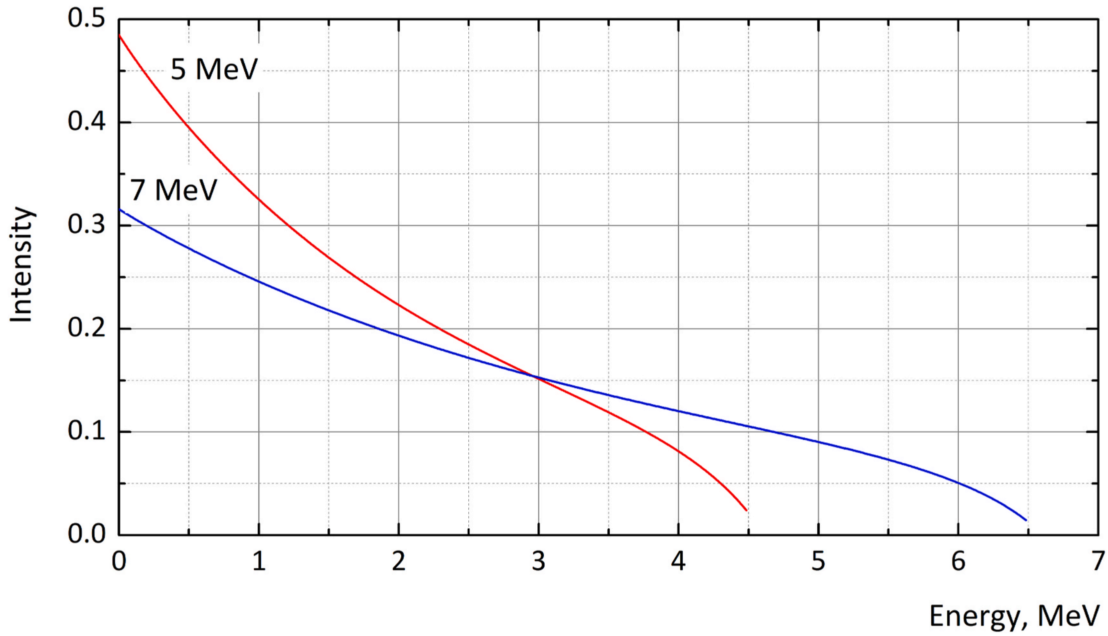

Fig. 1. The intensity of bremsstrahlung (normalized to 1). The total energy of electrons is 5 and 7 MeV, the material of the target is tantalum.

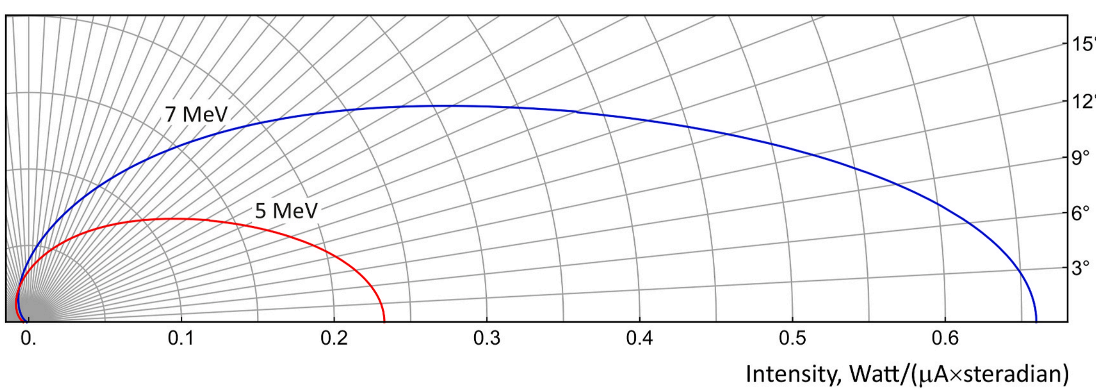

Fig. 2. Dependence of the bremsstrahlung power as a function of the photon emission angle. The total energy of electrons is 5 and 7 MeV, the material of the bremsstrahlung target is tantalum, the thickness of the bremsstrahlung target is 1.2 mm.

The energy of photons is transferred to matter through a two-step process. First, energy is transferred to secondary charged particles (electrons and positrons), then secondary charged particles transfer energy to matter through the excitation of atomic shells and ionization. To take into account all of the above, two more coefficients are introduced in radiation dosimetry: mass energy-transfer coefficient $\mu_{tr}(k)$ and mass energy-absorption coefficient $\mu_{en}(k)$ (Shani, 2001; Podgoršak-Agency, 2005). The mass energy-transfer coefficient is related to the mass attenuation coefficient as shown below:

$$\mu_{tr}(k) = \mu_m(k)\frac{E_{tr}}{k}, \tag{8}$$

where E_{tr}–is the average kinetic energy of secondary charged particles. Secondary charged particles, interacting with matter, lose their kinetic energy through ionization and radiation losses. Since the absorbed dose is determined by the ionization losses of secondary charged particles, the mass energy-absorption coefficient is explicitly used to calculate the absorbed dose:

$$\mu_{en}(k) = \mu_m(k)\frac{E_{en}}{k} = \mu_{tr}(1-g), \tag{9}$$

where E_{en}–is the average energy of secondary charged particles transferred to matter through ionization losses, g–is the averaged part of the kinetic energy of secondary charged particles converted into bremsstrahlung when passing through the matter. The NIST report (Hubbell and Seltzer, 1995) covers the photons energy range from 0.001 MeV to 20 MeV and provides values for 140 materials: the first 92 elements, from hydrogen through uranium, and 48 additional materials of dosimetric interest.

Substituting the expression for $\mu_m(k)$ from (7) into formula (9), we obtain:

$$\mu_{en}(k) = -\frac{1}{\rho\, N\, k}\frac{E_{en}dN}{dl} = -\frac{1}{\rho\, A}\frac{dA}{dl}, \tag{10}$$

where $A = N\,k$–is the energy of all photons incident normal to the material, $dA = E_{en}dN$– is the absorbed energy in the layer of matter of thickness dl. Separating the variable and integrating expression, we obtain:

$$A_{en} = A_0(1-\exp(-\mu_{en}(k)\rho\, l)), \tag{11}$$

where $A_0 = N_0\,k$– is the total energy of all photons, A_{en}–is the total absorbed energy.

Let the sample exposed to bremsstrahlung radiation be a rectangular

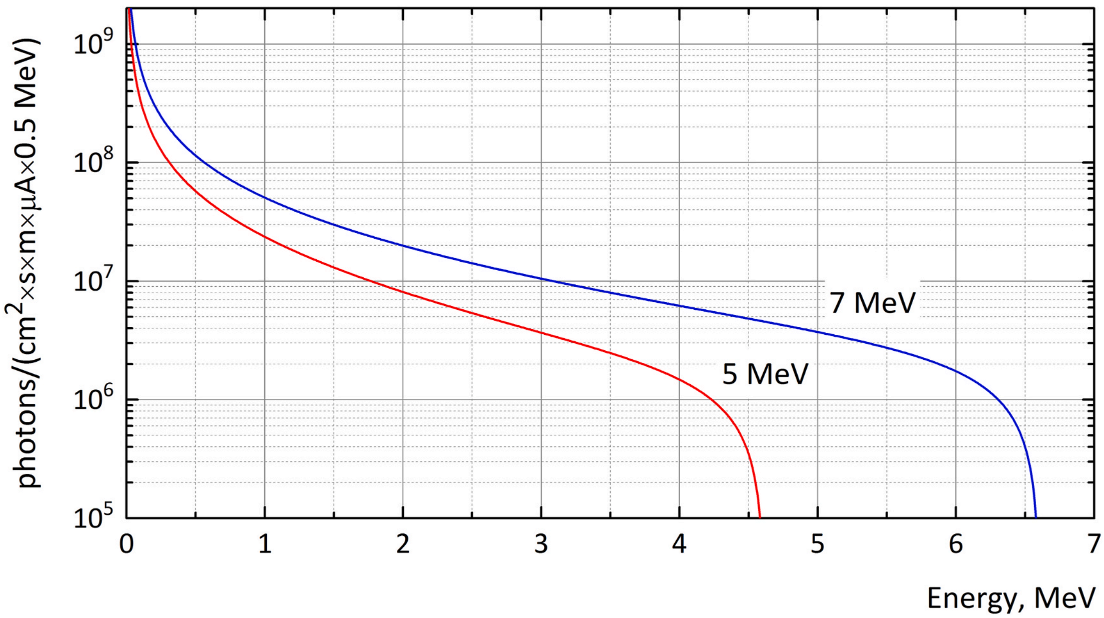

Fig. 3. Bremsstrahlung photon fluence (in the forward direction).

parallelepiped with edges a and b. The face of the parallelepiped, formed by edges a and b, with the area $S = ab$, is located perpendicular to the bremsstrahlung flux at a distance r from the bremsstrahlung target. The axis of symmetry of the parallelepiped is collinear to its larger side l and coincides with the direction of motion of the electrons generated by the accelerator (Fig. 4). In this case, the flux density of photons incident on the sample per unit time is determined by formula (6):

$$dN = \frac{W\,R(0)I(k)\,S}{k\,r^2}dk \tag{12}$$

Taking into account (11), the absorbed energy will be equal to:

$$dA_{en} = k\,dN(1 - \exp(-\mu_{en}(k)\rho l)) = \frac{W\,R(0)I(k)\,S}{r^2}(1 - \exp(-\mu_{en}(k)\rho l))dk \tag{13}$$

Integrating expression (13) over the entire bremsstrahlung spectrum, we find the total absorbed energy in the sample per unit time:

$$A_{en} = \frac{W\,R(0)\,S}{r^2} \times \int_0^{E_0 - m_e c^2} I(k) \times (1 - \exp(-\mu_{en}(k)\rho l))dk \tag{14}$$

Now we can find the value of the absorbed dose in the sample per unit time:

$$D = \frac{A_{en}}{m} = \frac{A_{en}}{\rho\,S\,l} \tag{15}$$

The exposure dose (in Roentgen units) can also be calculated using formulas (14, 15). An exposure dose of 1 Roentgen corresponds to an absorbed dose of 0.00877 Gy in dry air. Therefore, to calculate the exposure dose, it is sufficient to calculate the absorbed dose for an air cube with a volume of 1 cm^3.

For example, with electron energy of 5 MeV and a beam current of 1 μA for a sample representing a water cube with a volume of 1 cm^3 and located at a distance of 20 cm from a tantalum target with a thickness of 1.2 mm (see Fig. 4), the absorbed dose will be 0.022 Gy/s.

The results of calculations of the absorbed dose for electron energies of 5 and 7 MeV are in good agreement with calculations by the Monte Carlo method (Peri and Orion, 2017; Petwalet al., 2007).

5. Conclusion

The developed method for calculating the exposure and absorbed doses takes into account both the spectral composition of the bremsstrahlung and its angular anisotropy. The calculation of the absorbed dose can be performed for samples with arbitrary chemical composition and density. The algorithm is implemented in the Wolfram Mathematica and can be useful for specialists in the field of radiation materials science working with bremsstrahlung.

CRediT authorship contribution statement

Shakhboz Khasanov: Data curation, Writing Review & editing.
Renat Suleymanov: Software, Writing - original draft, investigation.
Akmal Safarov: Resources, Data curation.
Askar Safarov: Supervision, Methodology.

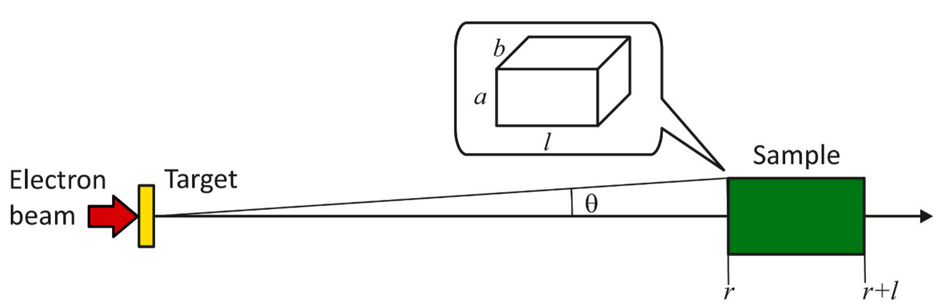

Fig. 4. Irradiation geometry.

Habtamu Menberu Tedila: Review & Validation.
Ruslan Muratov: Formal analysis, Software.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix A. Supplementary data

Supplementary data to this article can be found online at https://doi.org/10.1016/j.radphyschem.2021.109651.

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