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On centrality and rapidity dependences of transverse momentum spectra of negative pions in ${}^{12}C + {}^{12}C$ collisions at $4.2 \,\text{GeV}/c$ per nucleon

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The dependences of the experimental transverse momentum spectra of the negative pions, produced in minimum bias ${}^{12}C + {}^{12}C$ collisions at a momentum of $4.2A \,\text{GeV}/c$,

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on the collision centrality and the pion rapidity range were studied. To analyze quantitatively the change in the p_t spectra of π^- mesons with the changes of collision centrality and pion rapidity range, the extracted p_t spectra were fitted by Hagedorn, Boltzmann, Simple Exponential and Gaussian functions. The values of the extracted spectral temperatures T_1 and T_2 were consistently larger for the p_t spectra of π^- mesons coming from midrapidity range as compared to those of the negative pions generated in the target and projectile fragmentation regions. The spectral temperatures T_1 and T_2 extracted from fitting the p_t spectra of π^- mesons in range $p_t = 0.1 - 1.2 \, \text{GeV}/c$ practically coincided with each other in peripheral, semicentral and central ${}^{12}C + {}^{12}C$ collision events, and thus did not show any collision centrality dependence. However, the values of T_1 and T_2 extracted from fitting in range $p_t = 0.1-0.7 \, \text{GeV}/c$ were consistently and noticeably larger in case of central collisions as compared to peripheral and semicentral ${}^{12}C + {}^{12}C$ collisions. Hagedorn and Boltzmann functions provided significantly better fits of the transverse momentum spectra of the negative pions with the physically acceptable values of the extracted temperatures as compared to Gaussian and Simple Exponential functions.

Keywords: Relativistic nucleus–nucleus collisions; transverse momentum distribution of hadrons; pions; spectral temperatures of hadrons; Hagedorn thermodynamic model; Boltzmann function.

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1. Introduction

It is a well-known fact that pions are most abundantly produced particles in relativistic nuclear collisions and thus may carry useful information on the collision dynamics. The negative pions can be unambiguously separated from the other products of nuclear collisions. The production of such particles in hadron–nucleus and nucleus–nucleus collisions at the energies of the order of few GeV/nucleon is mostly due to excitation of baryon resonances decaying finally into nucleons and pions. A significant fraction of the pions produced in relativistic hadron–nucleus and nucleus–nucleus collisions in bubble chamber experiments of Joint Institute for Nuclear Research (JINR, Dubna, Russia) was shown to come from decay of Δ 's in earlier works.¹⁻⁹ That the 8 resonances play a significant role in pion production at the energies of the order of a few GeV/nucleon was also stated in Refs. 10–14.

The spectral temperatures of π^- mesons produced in $d + {}^{12}\text{C}$, ${}^{4}\text{He} + {}^{12}\text{C}$ and ${}^{12}\text{C} + {}^{12}\text{C}$ collisions at $4.2A \,\text{GeV}/c$ were extracted by fitting noninvariant centerof-mass (cm) energy spectra of π^- mesons with Maxwell–Boltzmann distribution function in Ref. 15. The rapidity and angular dependences of spectral temperatures of the negative pions produced in ${}^{12}\text{C} + {}^{12}\text{C}$ collisions at $4.2A \,\text{GeV}/c$ were analyzed in Ref. 16. The temperatures of the negative pions were estimated and studied for various sets of colliding nuclei at different energies. ${}^{15,17-21}$ Transverse momentum and transverse mass distributions are preferred for extracting the hadron temperatures due to their Lorentz invariance with respect to longitudinal boosts. 17,20,22,23 Transverse momentum spectra are assumed not to be affected by the longitudinal collective motion compared to the energy spectra of hadrons. This work is continuation of our recent papers^{16,23,24} on analysis of various features of the negative pions produced in nucleus–nucleus collisions at 4.2A GeV/c. The goal of this paper is to study the dependences of the experimental p_t spectra of the negative pions, produced in ${}^{12}\text{C} + {}^{12}\text{C}$ collisions at a momentum of 4.2 GeV/cper nucleon, on the collision centrality and on the range of π^- rapidity. To analyze quantitatively the influence of the change of the collision centrality and pion rapidity range on the shape of the p_t spectra of π^- in ${}^{12}\text{C} + {}^{12}\text{C}$ collisions, the extracted p_t spectra were fitted by four of the functions commonly used for quantitative analysis of the hadron spectra. It is also interesting to check which of these functions are more appropriate for fitting the p_t spectra of π^- mesons in ${}^{12}\text{C} + {}^{12}\text{C}$ collisions.

2. Experimental Procedures and Analysis

The data analyzed in the present work were obtained using the 2 m propane (C₃H₈) bubble chamber of Laboratory of High Energies of JINR (Dubna, Russia). The 2 m propane bubble chamber was placed in a magnetic field of strength $1.5 \, \mathrm{T}.^{25-32}$ The bubble chamber was irradiated with beams of 12 C nuclei accelerated to a momentum of $4.2 \,\text{GeV}/c$ per nucleon at Dubna synchrophasotron. Methods of selection of inelastic ${}^{12}C + {}^{12}C$ collision events in this experiment were given in detail in Refs. 25–32. Threshold for detection of π^- mesons produced in ${}^{12}C + {}^{12}C$ collisions was around $70 \,\mathrm{MeV}/c$. In some momentum and angular intervals, the particles could not be detected with 100% efficiency. To account for small losses of particles emitted under large angles to object plane of the camera, relevant corrections were introduced as discussed in Refs. 25–32. The average uncertainty in measurement of emission angle of the negative pions was 0.8° . The mean relative uncertainty of momentum measurement of π^- mesons from the curvature of their tracks in propane bubble chamber was 6%. All the negative particles, except those identified as electrons, were considered to be π^- mesons. Admixtures of unidentified electrons and negative strange particles among them were less than 5%. In our experiment, the spectator protons are protons with momenta $p > 3 \,\mathrm{GeV}/c$ and emission angle $\theta < 4^{\circ}$ (projectile spectators), and protons with momenta $p < 0.3 \,\mathrm{GeV}/c$ (target spectators) in the laboratory frame.²⁵⁻³² Thus, the participant protons are the protons which remain after elimination of the spectator protons. Statistics of the experimental data analyzed in the present work consist of 20,528 $^{12}C + ^{12}C$ minimum bias inelastic collision events with almost all the secondary charged particles detected and measured with 4π acceptance.

Comparison of the mean multiplicities per event of the negative pions and participant protons and the average values of rapidity and transverse momentum of $\pi^$ mesons in ${}^{12}\text{C} + {}^{12}\text{C}$ collisions at $4.2 \,\text{GeV}/c$ per nucleon both in the experiment and Quark–Gluon-String-Model (QGSM)^{33–36} is presented in Table 1. QGSM was developed to describe hadron–nucleus and nucleus–nucleus collisions at high energies. In the QGSM, hadron production occurs via formation and decay of quark–gluon strings. This model is used as a basic process for generation of hadron–hadron

Table 1. Mean multiplicities per event of the negative pions and participant protons and the average values of rapidity and transverse momentum of π^- mesons in ${}^{12}\text{C} + {}^{12}\text{C}$ collisions at $4.2 \,\text{GeV}/c$ per nucleon. The mean rapidities are calculated in cm of nucleon-nucleon collisions at $4.2 \,\text{GeV}/c$. Only statistical errors are given here and at tables that follow.

Type	$\langle n(\pi^-) \rangle$	$\langle n_{\rm part.prot.} \rangle$	$\langle y_{ m cm}(\pi^-) \rangle$	$\langle p_t(\pi^-) \rangle ~(\text{GeV}/c)$
Exper QGSM	1.45 ± 0.01 1.59 ± 0.01	4.35 ± 0.02 4.00 ± 0.02	$\begin{array}{c} -0.016 \pm 0.005 \\ 0.007 \pm 0.005 \end{array}$	$\begin{array}{c} 0.242 \pm 0.001 \\ 0.219 \pm 0.001 \end{array}$

collisions. In the present work, the version of QGSM³⁴ adapted to the range of intermediate energies ($\sqrt{S_{nn}} \leq 4 \text{ GeV}$) was used. The incident momentum of 4.2 GeV/c per nucleon for ¹²C + ¹²C collisions analyzed in the present work corresponds to incident kinetic energy 3.37 GeV per nucleon and nucleon–nucleon cm energy $\sqrt{S_{nn}} = 3.14 \text{ GeV}$. The QGSM is based on Regge and string phenomenology of particle production in inelastic binary hadron collisions. To describe the evolution of the hadron and quark–gluon phases, a coupled system of Boltzmann-like kinetic equations was used in the model. The nuclear collisions were treated as a superposition of independent interactions of the projectile and target nucleons, stable hadrons and short-lived resonances. Resonant reactions like $\pi + N \rightarrow \Delta$, pion absorption by *NN* quasi-deuteron pairs and also $\pi + \pi \rightarrow \rho$ were taken into account in this model. The time of formation of hadrons was also included in QGSM. The masses of strings at intermediate energies are very small. At cm energy $\sqrt{S_{nn}} = 3.14 \text{ GeV}$, the masses of strings are smaller than 2 GeV, and these strings fragment predominantly (~90%) through two-particle decay channel.

The total transverse momentum and rapidity distributions of the negative pions in minimum bias ${}^{12}\text{C} + {}^{12}\text{C}$ collisions at a momentum of $4.2 \,\text{GeV}/c$ per nucleon are presented in Fig. 1. As can be seen from Fig. 1(a), the experimental transverse momentum spectrum of π^- mesons is described satisfactorily by the QGSM calculations. However, Fig. 1(a) shows that the QGSM underestimates the experimental p_t spectrum of π^- mesons in region $p_t > 0.7 \,\mathrm{GeV}/c$. It is important to mention that another model — Modified FRITIOF model, $^{26,37-39}$ designed also for describing the nucleus–nucleus collisions at incident energies of the order of a few GeV per nucleon, also underestimates this high p_t part of the pion spectra.^{23,40} In Ref. 23 it was observed that the fitting of the p_t spectra of π^- in $d + {}^{12}C$, ${}^{4}He + {}^{12}C$ and $^{12}C + ^{12}C$ collisions at $4.2A \,\text{GeV}/c$ with the two-temperature Hagedorn function resulted in the lower spectral temperatures T_1 and T_2 for both QGSM and Modified FRITIOF model spectra as compared to the experimental p_t distributions. It is also evident from Fig. 1(a) that the inverse slope of the QGSM spectrum is noticeably lesser compared to that of the experimental p_t spectrum. The rapidity spectrum in Fig. 1(b) is plotted in cm of nucleon–nucleon collisions at $4.2 \,\text{GeV}/c$ (the rapidity of the cm of nucleon–nucleon collision is $y_{\rm cm} \approx 1.1$ at this incident momentum). As observed from Fig. 1(b), QGSM describes quite well the experimental rapidity spectrum of π^- mesons in ${}^{12}C + {}^{12}C$ collisions.

On centrality and rapidity dependences of transverse momentum spectra



Fig. 1. The experimental transverse momentum (a) and rapidity (b) spectra of the negative pions produced in minimum bias ${}^{12}C + {}^{12}C$ (•) collisions at 4.2 GeV/c per nucleon. The corresponding calculated QGSM spectra are given by the solid lines. All the spectra are normalized per one inelastic collision event.

In the present work, the transverse momentum spectra of π^- mesons produced in ${}^{12}\text{C} + {}^{12}\text{C}$ collisions at a momentum of $4.2 \,\text{GeV}/c$ per nucleon were fitted by four different functions commonly used for describing the p_t spectra of hadrons. The Hagedorn thermodynamic model^{22,41} predicts that the normalized transverse momentum (p_t) distribution of hadrons can be described using the expression

$$\frac{dN}{Np_t dp_t} = A \cdot (m_t T)^{1/2} \exp\left(-\frac{m_t}{T}\right),\tag{1}$$

where N (depending on the choice of normalization) is either the total number of inelastic events or the total number of the respective hadrons, $m_t = \sqrt{m^2 + p_t^2}$ is the transverse mass, T is the spectral temperature and A is the fitting constant. This relation (1) will be referred to as the one-temperature Hagedorn function in this paper. In case of two temperatures, T_1 and T_2 , the above formula reads as

$$\frac{dN}{Np_t dp_t} = A_1 \cdot (m_t T_1)^{1/2} \exp\left(-\frac{m_t}{T_1}\right) + A_2 \cdot (m_t T_2)^{1/2} \exp\left(-\frac{m_t}{T_2}\right), \qquad (2)$$

referred to as the two-temperature Hagedorn function in this work. The above expressions (1) and (2) were derived assuming that $m_t \gg T$.

According to Boltzmann model, the transverse momentum spectra of hadrons can be fitted using m_t Boltzmann distribution function given by

$$\frac{dN}{Np_t dp_t} = Am_t \exp\left(-\frac{m_t}{T}\right),\tag{3}$$

referred to as the one-temperature Boltzmann function in the present work. In case of two temperatures, T_1 and T_2 , the above formula is modified as

$$\frac{dN}{Np_t dp_t} = A_1 \cdot m_t \exp\left(-\frac{m_t}{T_1}\right) + A_2 \cdot m_t \exp\left(-\frac{m_t}{T_2}\right).$$
(4)

The spectra of pions can also be fitted by Simple Exponential function as follows for the one-temperature and two-temperature scenarios, respectively:

$$\frac{dN}{Np_t dp_t} = A \exp\left(-\frac{p_t}{T}\right) \tag{5}$$

and

$$\frac{dN}{Np_t dp_t} = A_1 \cdot \exp\left(-\frac{p_t}{T_1}\right) + A_2 \cdot \exp\left(-\frac{p_t}{T_2}\right).$$
(6)

Another possibility for fitting the transverse momentum spectra of hadrons could be the Gaussian function given below for the one-temperature and the twotemperature cases as

$$\frac{dN}{Np_t dp_t} = A \exp\left(-\frac{p_t^2}{T^2}\right) \tag{7}$$

and

$$\frac{dN}{Np_t dp_t} = A_1 \cdot \exp\left(-\frac{p_t^2}{T_1^2}\right) + A_2 \cdot \exp\left(-\frac{p_t^2}{T_2^2}\right),\tag{8}$$

respectively.

We fitted the total transverse momentum spectra of the negative pions in the whole p_t range in ${}^{12}C + {}^{12}C$ collisions at 4.2A GeV/c by the two-temperature and the one-temperature Hagedorn and Boltzmann functions. The experimental p_t spectra of the negative pions produced in minimum bias ${}^{12}C + {}^{12}C$ collisions at $4.2A \,\text{GeV}/c$ per nucleon and the respective fits in the whole p_t range by the one-temperature and the two-temperature Hagedorn functions are presented in Fig. 2. As can be seen from Fig. 2, the two-temperature Hagedorn function describes the total p_t spectra of the negative pions very well and much better compared to the one-temperature fit. The parameters extracted from fitting the total transverse momentum spectra of the negative pions in the whole range of p_t in ${}^{12}C + {}^{12}C$ collisions at 4.2A GeV/c by the two-temperature and the one-temperature Hagedorn and Boltzmann functions are shown in Table 2. As can be seen from comparison of $\chi^2/n.d.f.$ and R^2 factor values in Table 2, the two-temperature Hagedorn and the two-temperature Boltzmann function fits describe the experimental spectra much better compared to the corresponding one-temperature fits. This result is in line with our recent papers^{16,23} and earlier works,^{15,17,19,20} where the transverse momentum as well as energy spectra of pions, produced in relativistic nuclear collisions, were characterized by the two-temperature shapes. In Ref. 17, the two-temperature shape of cm kinetic energy spectra of the negative pions in Ar + KCl collisions at $1.8 \, GeV/nucleon$ was revealed. In this work, the occurrence of two temperatures, T_1 and T_2 , was explained as due



Fig. 2. The experimental transverse momentum spectra (•) of the negative pions produced in minimum bias ${}^{12}C + {}^{12}C$ collisions at $4.2 \,\text{GeV}/c$ per nucleon and the corresponding fits in the whole p_t range by the one-temperature (dashed line) and the two-temperature (solid line) Boltzmann functions. All the spectra are normalized per one inelastic collision event.

Table 2.	The parameters	extracted from	n fitting the to	tal transverse	momentum	spectra	of the
negative	pions in the whole	p_t range in ¹² ($C + {}^{12}C$ collision	ns at $4.2A \mathrm{GeV}$	V/c by the tw	vo-tempe	rature
and the	one-temperature H	Iagedorn and E	Boltzmann func	tions.			

Fitting function	$A_1 \; (\text{GeV})^{-1}$	T_1 (MeV)	$A_2 \; (\text{GeV})^{-1}$	$T_2 (MeV)$	$\chi 2/{\rm n.d.f.}$	R^2 factor
Two-temperature Hagedorn	3097 ± 353	76 ± 3	101 ± 38	142 ± 7	1.33	0.99
One-temperature Hagedorn	1355 ± 74	99 ± 1			8.17	0.94
Two-temperature Boltzmann	3088 ± 326	65 ± 2	95 ± 26	127 ± 5	1.40	0.99
One-temperature Boltzmann	1140 ± 62	89 ± 1		—	11.83	0.92

to two channels of pion production: pions coming from Δ resonance decay (T_1) and directly produced pions (T_2) . In Ref. 19 the two-temperature shape of kinetic energy spectrum of pions emitted at 90° in cm of central La + La collisions at 1.35 GeV/nucleon was interpreted as due to different contributions of deltas originated from the early and later stages of heavy-ion reactions. The two-temperature behavior was also observed for cm energy as well as the p_t spectra of π^- mesons produced in Mg + Mg collisions²⁰ at incident momentum of 4.2–4.3A GeV/c. In Ref. 15, the two-temperature shape of the experimental cm energy spectra of $\pi^$ mesons in ${}^{12}C+{}^{12}C$ collisions at 4.2A GeV/c was explained by superposition of partial contributions of different sources (decays of resonances, direct reactions, etc.), analyzing the spectra of π^- mesons coming from different sources in the framework of QGSM.

It would be too naive to believe that the origin of pions in a minimum bias sample of ${}^{12}C + {}^{12}C$ collisions could only be described by two thermal sources. The phenomenon of collective flow has become the well-established and an important feature of relativistic heavy-ion collisions. Inverse slope parameter, T, or an apparent temperature of the emitting source, of transverse mass spectra of hadrons was shown to consist of two components: a thermal part, T_{thermal} , and a second part resembling the collective expansion with an average transverse velocity $\langle \beta_t \rangle$.⁴² Hence, the observed two-temperature shape of the transverse momentum spectrum of the negative pions produced in ${}^{12}C + {}^{12}C$ collisions at $4.2A \,\text{GeV}/c$ could also be interpreted by the collective flow effects.

It is important to add that in Ref. 18, the spectral temperature $T = 98 \pm 2 \text{ MeV}$ was extracted from fitting the p_t spectra of the negative pions in ${}^{4}\text{He} + {}^{12}\text{C}$ collisions at 4.5A GeV/c by the one-temperature Hagedorn function. This value of T practically coincides with the spectral temperature $T = 99 \pm 1 \text{ MeV}$ extracted from fitting the p_t spectra of π^- mesons in ${}^{12}\text{C} + {}^{12}\text{C}$ collisions at 4.2A GeV/c by the same one-temperature function and presented in Table 2.

The spectral temperatures (T_1, T_2) of π^- mesons in ${}^{12}C + {}^{12}C$ collisions at 4.2A GeV/c and their relative contributions (R_1, R_2) extracted in the present work from fitting the p_t spectra by the two-temperature Hagedorn and the two-temperature Boltzmann functions are given in Table 3. The corresponding results obtained in Ref. 15 from fitting the noninvariant cm energy spectra of the negative pions in ${}^{12}C + {}^{12}C$ collisions at the same initial momentum using two-temperature Maxwell–Boltzmann distribution function are also given for a comparison in this table. The relative contributions, R, of the two temperatures to the total negative pion multiplicity were calculated over the total transverse momentum interval:

$$R_i = c_i/(c_1 + c_2),$$

where $c_i = A_i \cdot \int (m_t T_i)^{1/2} \exp(-\frac{m_t}{T_i}) dp_t$ and $c_i = A_i \cdot \int m_t \exp(-\frac{m_t}{T_i}) dp_t (i = 1, 2)$ are for the case of Hagedorn and Boltzmann function fits, respectively. It is necessary to mention that the statistics of ${}^{12}\text{C} + {}^{12}\text{C}$ collisions used in Ref. 15 was 6806 inelastic collision events, which is about three times lesser compared to the statistics used in the present work. As observed from Table 3, the values of the spectral temperatures (T_1, T_2) extracted in the present work from fitting the p_t

Table 3. The spectral temperatures (T) of the negative pions in ${}^{12}\text{C} + {}^{12}\text{C}$ collisions at 4.2A GeV/c and their relative contributions (R) extracted in the present work from fitting their total transverse momentum spectra in the whole range of p_t by the two-temperature Hagedorn and Boltzmann functions compared to the corresponding values obtained in Ref. 15 from fitting the noninvariant cm energy spectra of the negative pions in ${}^{12}\text{C} + {}^{12}\text{C}$ collisions at 4.2A GeV/c by Maxwell–Boltzmann distribution function.

Fitting function	$T_1 (MeV)$	R_1 (%)	$T_2 \ ({\rm MeV})$	$R_2 \ (\%)$	$\chi 2/n.d.f.$	R^2 factor
Hagedorn Boltzmann	76 ± 3 65 ± 2	85 ± 14 85 ± 13	142 ± 7 127 ± 5	15 ± 6 15 ± 4	$1.32 \\ 1.40$	$0.99 \\ 0.99$
Maxwell–Boltzmann	83 ± 3	79 ± 6	145 ± 7	21 ± 6	0.72	_

spectra by the two-temperature Hagedorn and the two-temperature Boltzmann functions proved to be noticeably lower compared to the corresponding values obtained in Ref. 15 from fitting the noninvariant cm energy spectra of the negative pions by Maxwell–Boltzmann distribution for ${}^{12}C + {}^{12}C$ collisions at 4.2A GeV/c. As observed from Table 3, the dominant contribution $(R_1 \sim 85\%)$ to the total π^- multiplicity in ${}^{12}C + {}^{12}C$ collisions is given by $T_1 \sim (65-76) \pm 3 \text{ MeV}$, which is compatible within uncertainties with the results of Ref. 15. It is necessary to note that the fits by Boltzmann function result in somewhat lower values of the spectral temperatures T_1 and T_2 compared to those by Hagedorn function. However, as seen from Table 3, the values of the relative contributions $(R_1 \text{ and } R_2)$ for each temperature term coincided for both Hagedorn and Boltzmann function fits.

It is seen from Fig. 2 that the p_t spectrum of the negative pions with $p_t \leq$ $1.2 \,\mathrm{GeV}/c$ is characterized by sufficiently good statistics of π^- mesons, and therefore by quite low statistical errors. Due to the lower momentum threshold of detection of pions $p_{\rm thresh} \approx 70 \,{\rm MeV}/c$, it is natural to fit the transverse momentum spectra of the pions in range $p_t = 0.1 - 1.2 \,\text{GeV}/c$, where pions are detected and measured with almost 100% efficiency. The transverse momentum spectra of the negative pions in this p_t range in minimum bias ${}^{12}C + {}^{12}C$ collisions at $4.2A \,\text{GeV}/c$ were fitted by the two-temperature functions given in expressions (2), (4), (6) and (8). The parameters extracted from fitting the total transverse momentum spectra of the negative pions in range $p_t = 0.1-1.2 \,\mathrm{GeV}/c$ in ${}^{12}\mathrm{C}+{}^{12}\mathrm{C}$ collisions at $4.2A\,\mathrm{GeV}/c$ by these two-temperature functions are given in Table 4. As seen from Tables 2 and 4, the values of T_1 and T_2 extracted from fitting the p_t spectra of the negative pions in range $p_t = 0.1 - 1.2 \, \text{GeV}/c$ by the two-temperature Hagedorn and Boltzmann functions are noticeably lower than the corresponding temperature values extracted from fitting in the whole p_t range. This is likely due to the influence of the high p_t tail of the spectrum to the extracted values of the spectral temperatures in case of the fitting in the whole p_t range. It is seen from Table 4 that the fits by Hagedorn and Boltzmann functions result in physically acceptable values of T_1 and T_2 with quite small $\chi^2/n.d.f.$ values. The fitting with Simple Exponential function gives significantly larger values of T_1 and T_2 compared to the fits with Hagedorn and Boltzmann functions. The fitting with Gaussian function, as observed from

Table 4. The parameters extracted from fitting the total transverse momentum spectra of negative pions in the range $p_t = 0.1-1.2 \,\text{GeV}/c$ in ${}^{12}\text{C} + {}^{12}\text{C}$ collisions at $4.2A \,\text{GeV}/c$ by various two-temperature functions. (The units of A_1 and A_2 are $(\text{GeV})^{-1}$ in case of Hagedorn and Boltzmann function fits and dimensionless in case of Simple Exponential and Gaussian function fits wherever appropriate in the tables that follow.)

Fitting function	A_1	$T_1 \ (MeV)$	A_2	$T_2 \ ({\rm MeV})$	$\chi 2/{\rm n.d.f.}$	R^2 factor
Hagedorn	6464 ± 2250	59 ± 6	378 ± 110	119 ± 5	0.44	0.99
Boltzmann	5317 ± 1395	54 ± 4	252 ± 63	111 ± 4	0.42	0.99
Simple Exponential	105 ± 22	86 ± 19	44 ± 34	136 ± 14	0.61	0.99
Gaussian	4 ± 1	439 ± 8	45 ± 3	212 ± 6	1.74	0.99

Table 4, leads to very large and physically unacceptable values of T_1 and T_2 with relatively high $\chi^2/\text{n.d.f.}$ values.

The experimental transverse momentum spectrum of the negative pions produced in minimum bias ${}^{12}\text{C} + {}^{12}\text{C}$ collisions at $4.2 \,\text{GeV}/c$ per nucleon and the corresponding fits in the range $p_t = 0.1-1.2 \,\text{GeV}/c$ by the two-temperature Hagedorn and the two-temperature Boltzmann functions is presented in Fig. 3. As can be seen from Fig. 3, the two-temperature Hagedorn and the two-temperature Boltzmann functions fit very well the p_t spectrum of the negative pions in ${}^{12}\text{C} + {}^{12}\text{C}$ collisions.

It seems interesting to analyze quantitatively the change in the shape of transverse momentum spectra of the pions with increase in the collision centrality, which corresponds to decrease of the impact parameter of collision. Because the collision impact parameter is not directly measurable, we use the number of participant protons N_p to characterize the collision centrality. We follow Refs. 23, 25 and 43 to define peripheral collision events to be those in which $N_p \leq \langle n_{\text{part.prot.}} \rangle$, and central collisions as the collision events with $N_p \geq 2 \langle n_{\text{part.prot.}} \rangle$, where $\langle n_{\text{part.prot.}} \rangle$ is the mean multiplicity per event of participant protons, and semicentral collisions come in between these two multiplicity regions. It was shown earlier⁴³ that the central $^{12}\text{C} + ^{181}\text{Ta}$ collisions at 4.2A GeV/c selected using the above criterion were characterized by complete projectile stopping, because in these collisions the average number $\langle \nu^p \rangle$ of interacting projectile nucleons was very close to the total number of nucleons in projectile carbon. Fractions of central, semicentral and peripheral $^{12}\text{C} + ^{12}\text{C}$ collision events, relative to the total inelastic cross-section, obtained for both experimental and QGSM data are presented in Table 5. As seen from Table 5,



Fig. 3. The experimental transverse momentum spectra of the negative pions produced in minimum bias ${}^{12}\text{C} + {}^{12}\text{C}$ (•) collisions at $4.2 \,\text{GeV}/c$ per nucleon and the corresponding fits in the range $p_t = 0.1-1.2 \,\text{GeV}/c$ by the two-temperature Hagedorn (solid line) and the two-temperature Boltzmann (dashed line) functions. All the spectra are normalized per one inelastic collision event.

Peripheral col	lisions (%)	Semicentral co	ollisions (%)	Central collisions (%)		
Experiment	QGSM	Experiment	QGSM	Experiment	QGSM	
58 ± 1	62 ± 1	31 ± 1	30 ± 1	11 ± 1	8 ± 1	

Table 5. Fractions of central, semicentral and peripheral $^{12}\mathrm{C} + ^{12}\mathrm{C}$ collisions at $4.2\,\mathrm{GeV}/c$ per nucleon relative to the total inelastic cross-section.

the experimental and corresponding model fractions of peripheral, semicentral and central ${}^{12}C + {}^{12}C$ collision events coincide with each other within the two standard errors.

The p_t spectra of the negative pions in peripheral, semicentral and central $^{12}C + ^{12}C$ collision events in range $p_t = 0.1 - 1.2 \,\text{GeV}/c$ were fitted by the above two-temperature functions given in expressions (2), (4), (6) and (8). The parameters extracted from fitting the transverse momentum spectra of the negative pions in range $p_t = 0.1 - 1.2 \,\text{GeV}/c$ in peripheral, semicentral and central ${}^{12}\text{C} + {}^{12}\text{C}$ collisions at 4.2A GeV/c by various two-temperature functions are given in Table 6. As observed from Table 6, the values of T_1 and T_2 extracted from fitting the p_t spectra by the two-temperature Hagedorn and the two-temperature Boltzmann functions coincide with each other within the errors for peripheral, semicentral and central $^{12}C + ^{12}C$ collisions, and thus do not depend on the collision centrality. Such independence on the collision centrality of the spectral temperatures extracted from fitting in the whole range of the energy and p_t spectra of the negative pions by the two-temperature Maxwell–Boltzmann distribution and Hagedorn functions, respectively, in ${}^{12}C + {}^{12}C$ collisions at $4.2A \,\text{GeV}/c$ was also obtained earlier in Refs. 15 and 23. As observed from Table 6, the values of T_1 and T_2 extracted from fitting by Simple Exponential are significantly larger as compared to those obtained from

Fitting function	Collision type	A_1	$T_1 \ ({\rm MeV})$	A_2	T_2 (MeV)	$\chi 2/$ n.d.f.	R^2 factor
Hagedorn	Peripheral Semicentral Central	3437 ± 1415 9767 ± 3798 $12,800 \pm 5501$	58 ± 7 58 ± 6 62 ± 9	189 ± 82 526 ± 183 1013 ± 487	117 ± 7 121 ± 6 118 ± 8	$0.44 \\ 0.24 \\ 1.10$	$0.99 \\ 0.99 \\ 0.99 \\ 0.99$
Boltzmann	Peripheral Semicentral Central	$\begin{array}{c} 2864 \pm 903 \\ 8054 \pm 2381 \\ 11,114 \pm 3536 \end{array}$	$53 \pm 5 \\ 53 \pm 4 \\ 56 \pm 6$	$130 \pm 47 \\ 360 \pm 107 \\ 671 \pm 265$	108 ± 6 111 ± 5 109 ± 6	$\begin{array}{c} 0.42 \\ 0.25 \\ 1.05 \end{array}$	$0.99 \\ 0.99 \\ 0.99$
Simple exponential	Peripheral Semicentral Central	57 ± 16 159 ± 34 208 ± 173	88 ± 23 88 ± 20 91 ± 41	$17 \pm 25 \\ 53 \pm 54 \\ 121 \pm 223$	137 ± 26 140 ± 19 132 ± 28	$\begin{array}{c} 0.58 \\ 0.31 \\ 1.30 \end{array}$	$0.99 \\ 0.99 \\ 0.99 \\ 0.99$
Gaussian	Peripheral Semicentral Central	$2 \pm 1 \\ 6 \pm 1 \\ 11 \pm 1$	418 ± 10 432 ± 9 432 ± 11	$\begin{array}{c} 24\pm2\\ 67\pm5\\ 110\pm8 \end{array}$	201 ± 7 206 ± 6 212 ± 7	$1.02 \\ 1.49 \\ 1.23$	$0.99 \\ 0.99 \\ 0.99 \\ 0.99$

Table 6. The parameters extracted from fitting the transverse momentum spectra of the negative pions in the range $p_t = 0.1-1.2 \,\text{GeV}/c$ in peripheral, semicentral and central $^{12}\text{C} + ^{12}\text{C}$ collisions at $4.2A \,\text{GeV}/c$ by various two-temperature functions.

fitting by the two-temperature Hagedorn and the two-temperature Boltzmann functions. Moreover, as seen from Table 6, the fitting with Gaussian function leads to very large and physically unacceptable values of the spectral temperatures.

The experimental transverse momentum spectra of the negative pions produced in peripheral, semicentral and central ${}^{12}C + {}^{12}C$ collisions at 4.2 GeV/c per nucleon and the corresponding fits in range $p_t = 0.1-1.2 \text{ GeV}/c$ by the two-temperature Boltzmann function are given in Fig. 4. As seen from Fig. 4(a), the p_t spectra of the negative pions in central and semicentral ${}^{12}C + {}^{12}C$ collisions lie sufficiently above the corresponding spectrum for peripheral ${}^{12}C + {}^{12}C$ collisions. The obvious reason for this is that, with an increase in the collision centrality, the number of nucleon–nucleon collisions and, consequently, the number of the participant nucleons and produced pions increase. As can be seen from Fig. 4, the two-temperature Boltzmann function again fits very well the p_t spectra of the negative pions in peripheral, semicentral and central ${}^{12}C + {}^{12}C$ collisions.

It seems interesting also to analyze quantitatively the change in the shape of the p_t spectra of the negative pions with the change of the pion rapidity range. We extracted and fitted, using the above two-temperature functions, the transverse momentum spectra of π^- mesons for three different rapidity intervals in the nucleon-nucleon cm at $4.2 \,\text{GeV}/c$: $y_{\text{cm}} \leq -0.3$, $|y_{\text{cm}}| \leq 0.3$ and $y_{\text{cm}} \geq 0.3$, which can roughly be classified as target fragmentation, midrapidity and projectile fragmentation regions, respectively. The parameters extracted from fitting the transverse momentum spectra of the negative pions in range $p_t = 0.1-1.2 \,\text{GeV}/c$ in ${}^{12}\text{C} + {}^{12}\text{C}$ collisions at $4.2A \, \text{GeV}/c$ by the above-mentioned two-temperature functions for different pion rapidity intervals are presented in Table 7. It can be noted again that the fitting by Gaussian function results in large and physically unacceptable values of the spectral temperatures. As seen from Table 7, the absolute values of T_1 and T_2 were found to be noticeably larger for the midrapidity region than those for the target and projectile fragmentation regions for all the fitting functions, except for Simple Exponential function. This is in agreement with the results of the earlier works^{44,45}: with an increase in transverse momentum of π^- mesons in (p, d, α) (C) + C and $(d, \alpha, C) + Ta$ collisions at 4.2A GeV/c, the fraction of the negative pions in central rapidity region increased and the corresponding fraction in the fragmentation region of colliding nuclei decreased. Central rapidity (or midrapidity) interval was mostly populated with the negative pions with large transverse momenta.^{44,45} In recent work,¹⁶ we studied the rapidity and angular dependences of the average spectral temperatures of π^- mesons in ${}^{12}\text{C} + {}^{12}\text{C}$ collisions at $4.2A \,\text{GeV}/c$ by fitting their p_t spectra in the whole p_t range by the one-temperature Hagedorn function. The average spectral temperature of the negative pions extracted from p_t spectra increased from $36 \pm 2 \,\text{MeV}$ to $108 \pm 2 \,\text{MeV}$ when going from fragmentation region of colliding ¹²C nuclei to cm midrapidity region.¹⁶ The larger spectral temperatures T_1 and T_2 in cm midrapidity region compared to the target and projectile fragmentation regions observed in the present analysis are in agreement with our earlier finding for the average spectral temperatures.¹⁶ The reason for this is that



Fig. 4. The experimental transverse momentum spectra of the negative pions produced in peripheral (•) ((a) and (b)), semicentral (\blacktriangle) ((a) and (c)) and central (\blacksquare) ((a) and (d)) $^{12}C + ^{12}C$ collisions at $4.2 \,\mathrm{GeV}/c$ per nucleon and the corresponding fits in the range $p_t = 0.1 - 1.2 \,\mathrm{GeV}/c$ by the two-temperature Boltzmann function (solid lines). All the spectra are normalized per one inelastic collision event.

1.2

10-1

10-2

0.2

0.4

0.6

(d)

pt, GeV/c

0.8

1.0

1.2

10-1

10-2

0.2

0.4

0.6

(c)

pt, GeV/c

0.8

1.0

the pions in central rapidity region are produced predominantly in central hard $^{12}C + ^{12}C$ collisions, and hence at higher temperatures, as compared to the pions in region of fragmentation of colliding nuclei originated mostly in peripheral soft $^{12}\mathrm{C} + ^{12}\mathrm{C}$ interactions, and hence at lower temperatures. 16

Fitting function	Rapidity range	A_1	$T_1 (MeV)$	A_2	T_2 (MeV)	$\chi 2/$ n.d.f.	R^2 factor
Hagedorn	$y_{\rm cm} \le -0.3$ $ y_{\rm cm} \le 0.3$ $y_{\rm cm} \ge 0.3$	8463 ± 4016 1896 ± 506 $10,722 \pm 6542$	$53 \pm 6 \\ 78 \pm 10 \\ 49 \pm 8$	421 ± 131 137 ± 109 661 ± 213	113 ± 5 142 ± 15 102 ± 4	$0.83 \\ 0.47 \\ 1.04$	$0.99 \\ 0.99 \\ 0.99 \\ 0.99$
Boltzmann	$\begin{aligned} y_{\rm cm} &\leq -0.3 \\ y_{\rm cm} &\leq 0.3 \\ y_{\rm cm} &\geq 0.3 \end{aligned}$	6521 ± 2275 1842 ± 409 8094 ± 3502	$49 \pm 4 \\ 68 \pm 6 \\ 46 \pm 5$	$279 \pm 78 \\ 109 \pm 58 \\ 424 \pm 125$	$105 \pm 4 \\ 129 \pm 10 \\ 95 \pm 4$	$\begin{array}{c} 0.79 \\ 0.42 \\ 1.01 \end{array}$	$0.99 \\ 0.99 \\ 0.99 \\ 0.99$
Simple exponential	$\begin{array}{l} y_{\rm cm} \leq -0.3 \\ y_{\rm cm} \leq 0.3 \\ y_{\rm cm} \geq 0.3 \end{array}$	$\begin{array}{c} 95\pm25\\ 74\pm9\\ 80\pm33 \end{array}$	73 ± 22 122 ± 16 69 ± 35	53 ± 29 2 ± 14 78 ± 46	126 ± 10 212 ± 205 113 ± 9	$1.00 \\ 0.78 \\ 1.15$	$0.99 \\ 0.99 \\ 0.99 \\ 0.99$
Gaussian	$\begin{array}{l} y_{\rm cm} \leq -0.3 \\ y_{\rm cm} \leq 0.3 \\ y_{\rm cm} \geq 0.3 \end{array}$	$4 \pm 1 \\ 3 \pm 1 \\ 5 \pm 1$	413 ± 9 476 ± 14 384 ± 8	$\begin{array}{c} 44\pm 3\\ 28\pm 2\\ 47\pm 4\end{array}$	196 ± 6 235 ± 8 189 ± 6	$1.52 \\ 0.62 \\ 1.46$	$0.99 \\ 0.99 \\ 0.99 \\ 0.99$

Table 7. The parameters extracted from fitting the transverse momentum spectra of negative pions in the range $p_t = 0.1-1.2 \,\text{GeV}/c$ in ${}^{12}\text{C} + {}^{12}\text{C}$ collisions at $4.2A \,\text{GeV}/c$ by various two-temperature functions for different pion rapidity ranges.

The experimental transverse momentum spectra of the negative pions for the analyzed three rapidity intervals in ${}^{12}\text{C} + {}^{12}\text{C}$ collisions at $4.2A \,\text{GeV}/c$ per nucleon and the corresponding fits in range $p_t = 0.1-1.2 \,\text{GeV}/c$ by the two-temperature Hagedorn function are presented in Fig. 5. As observed from Fig. 5, the p_t spectra of the negative pions in peripheral, semicentral and central ${}^{12}\text{C} + {}^{12}\text{C}$ collisions are described very well by the two-temperature Hagedorn function.

To check the influence of the fitting range of p_t on the extracted values of T_1 and T_2 , the total p_t spectra of the negative pions and the respective spectra of π^- mesons for different ${}^{12}C + {}^{12}C$ collision centralities and three rapidity regions were also fitted in range $p_t = 0.1-0.7 \text{ GeV}/c$. The parameters extracted from fitting the total p_t spectra of π^- mesons in range $p_t = 0.1-0.7 \text{ GeV}/c$ in ${}^{12}C + {}^{12}C$ collisions at 4.2A GeV/c by the considered two-temperature functions are presented in Table 8. It is seen from comparison of Tables 4 and 8 that the values of the spectral temperatures T_1 and T_2 are compatible within the uncertainties for the fitting ranges $p_t = 0.1-0.7 \text{ GeV}/c$ and $p_t = 0.1-1.2 \text{ GeV}/c$ for the case of Hagedorn and Boltzmann function fits. One can see from Table 8 that the values of the spectral temperatures extracted from fitting with the two-temperature Simple Exponential and the two-temperature Gaussian functions are again too large to be acceptable for the colliding system and collision energy under consideration.

The parameters extracted from fitting the transverse momentum spectra of the negative pions in the range $p_t = 0.1-0.7 \,\text{GeV}/c$ in peripheral, semicentral and central $^{12}\text{C} + ^{12}\text{C}$ collisions at $4.2A \,\text{GeV}/c$ by the above given two-temperature functions are presented in Table 9. As observed from Table 9, the absolute values of the spectral temperatures T_1 and T_2 for central $^{12}\text{C} + ^{12}\text{C}$ collisions were consistently higher as compared to the corresponding temperatures for peripheral and semicentral collisions.



Fig. 5. The experimental transverse momentum spectra of the negative pions for rapidity range $y_{\rm cm} \leq -0.3$ (•) ((a) and (b)), for rapidity range $|y_{\rm cm}| \leq 0.3$ (•) ((a) and (c)), and for rapidity range $y_{\rm cm} \geq 0.3$ (•) ((a) and (d)) in ${}^{12}{\rm C} + {}^{12}{\rm C}$ collisions at $4.2 \,{\rm GeV}/c$ per nucleon and the corresponding fits in the range $p_t = 0.1-1.2 \,{\rm GeV}/c$ by the two-temperature Hagedorn function (solid lines). All the spectra are normalized per one negative pion.

Table 8. The parameters extracted from fitting the total transverse momentum spectra of the negative pions in the range $p_t = 0.1-0.7 \text{ GeV}/c$ in ${}^{12}\text{C} + {}^{12}\text{C}$ collisions at 4.2A GeV/c by various two-temperature functions.

Fitting function	A_1	$T_1 (MeV)$	A_2	$T_2 \ ({\rm MeV})$	$\chi 2/{\rm n.d.f.}$	R^2 factor
Hagedorn	5461 ± 2189	64 ± 10	216 ± 259	131 ± 26	0.35	0.99
Boltzmann	5006 ± 1679	56 ± 7	201 ± 162	115 ± 15	0.33	0.99
Simple exponential	134 ± 15	105 ± 10	0.573 ± 3.4	499 ± 199	0.53	0.99
Gaussian	7 ± 1	386 ± 15	49 ± 4	191 ± 9	0.70	0.99

Table 9. The parameters extracted from fitting the transverse momentum spectra of the negative pions in the range $p_t = 0.1-0.7 \,\text{GeV}/c$ in peripheral, semicentral and central $^{12}\text{C} + ^{12}\text{C}$ collisions at $4.2A \,\text{GeV}/c$ by various two-temperature functions.

Fitting function	Collision type	A_1	$T_1 \ ({\rm MeV})$	A_2	$T_2 (MeV)$	$\chi 2/$ n.d.f.	R^2 factor
Hagedorn	Peripheral Semicentral Central	2735 ± 1127 $10,156 \pm 5741$ 9194 ± 2986	$65 \pm 11 \\ 58 \pm 11 \\ 74 \pm 10$	79 ± 130 531 ± 480 116 ± 325	136 ± 39 121 ± 18 174 ± 103	$\begin{array}{c} 0.31 \\ 0.38 \\ 0.76 \end{array}$	$0.99 \\ 0.99 \\ 0.99 \\ 0.99$
Boltzmann	Peripheral Semicentral Central	2531 ± 904 8895 ± 3931 8838 ± 2587	56 ± 7 52 ± 7 63 ± 8	84 ± 90 421 ± 285 204 ± 334	116 ± 21 109 ± 12 135 ± 42	$\begin{array}{c} 0.31 \\ 0.35 \\ 0.76 \end{array}$	$0.99 \\ 0.99 \\ 0.99$
Simple exponential	Peripheral Semicentral Central	60 ± 4 191 ± 39 286 ± 19	110 ± 2 98 ± 27 115 ± 2	12 ± 67	185 ± 216	$1.06 \\ 0.55 \\ 1.18$	$0.99 \\ 0.99 \\ 0.99$
Gaussian	Peripheral Semicentral Central	$\begin{array}{c} 4\pm1\\ 12\pm2\\ 16\pm4 \end{array}$	378 ± 19 377 ± 15 399 ± 26	$\begin{array}{c} 25\pm2\\ 72\pm7\\ 112\pm10 \end{array}$	187 ± 10 184 ± 9 202 ± 11	$\begin{array}{c} 0.62 \\ 0.46 \\ 1.03 \end{array}$	$0.99 \\ 0.99 \\ 0.99 \\ 0.99$

Table 10. The parameters extracted from fitting the transverse momentum spectra of the negative pions in the range $p_t = 0.1-0.7 \,\text{GeV}/c$ in ${}^{12}\text{C} + {}^{12}\text{C}$ collisions at $4.2A \,\text{GeV}/c$ by various two-temperature functions for different pion rapidity ranges.

Fitting function	Rapidity range	A_1	$T_1 (MeV)$	A_2	$T_2 ({\rm MeV})$	$\chi 2/$ n.d.f.	R^2 factor
Hagedorn	$y_{\rm cm} \le -0.3$ $ y_{\rm cm} \le 0.3$ $y_{\rm cm} \ge 0.3$	6639 ± 3151 1155 ± 96 7735 ± 4112	59 ± 9 101 ± 2 56 ± 10	224 ± 232 $$ 349 ± 354	126 ± 22 $$ 114 ± 18	$0.40 \\ 1.63 \\ 0.71$	$0.99 \\ 0.99 \\ 0.99 \\ 0.99$
Boltzmann	$\begin{array}{l} y_{\rm cm} \leq -0.3 \\ y_{\rm cm} \leq 0.3 \\ y_{\rm cm} \geq 0.3 \end{array}$	5780 ± 2241 1020 ± 79 6636 ± 2827	$52 \pm 6 \\ 89 \pm 1 \\ 50 \pm 6$	193 ± 146 275 ± 207	112 ± 15 102 ± 12	$\begin{array}{c} 0.37 \\ 2.87 \\ 0.69 \end{array}$	$0.99 \\ 0.98 \\ 0.99$
Simple exponential	$\begin{array}{l} y_{\rm cm} \leq -0.3 \\ y_{\rm cm} \leq 0.3 \\ y_{\rm cm} \geq 0.3 \end{array}$	$131 \pm 14 \\ 77 \pm 5 \\ 140 \pm 17$	97 ± 15 128 ± 3 93 ± 20	3 ± 13 $$ 6 ± 33	238 ± 316 $$ 180 ± 221	$0.62 \\ 0.78 \\ 0.87$	$0.99 \\ 0.99 \\ 0.99 \\ 0.99$
Gaussian	$\begin{array}{l} y_{\rm cm} \leq -0.3 \\ y_{\rm cm} \leq 0.3 \\ y_{\rm cm} \geq 0.3 \end{array}$	7 ± 1 5 ± 2 7 ± 1	$378 \pm 16 \\ 423 \pm 36 \\ 363 \pm 15$	$\begin{array}{c} 47\pm 4\\ 30\pm 2\\ 49\pm 4 \end{array}$	$\begin{array}{c} 182 \pm 9 \\ 220 \pm 14 \\ 181 \pm 8 \end{array}$	$\begin{array}{c} 0.49 \\ 0.48 \\ 0.94 \end{array}$	$0.99 \\ 0.99 \\ 0.99 \\ 0.99$

The parameters extracted from fitting the transverse momentum spectra of the negative pions in range $p_t = 0.1-0.7 \text{ GeV}/c$ in ${}^{12}\text{C} + {}^{12}\text{C}$ collisions at 4.2A GeV/c by the same two-temperature functions for different pion rapidity ranges are given in Table 10. The fitting of the p_t spectra of π^- mesons coming from midrapidity range ($|y_{cm}| \leq 0.3$) with the two-temperature Hagedorn, Boltzmann and Simple Exponential functions resulted in the negative values of the parameter A_2 with the extracted temperatures T_2 practically coinciding with the corresponding values of T_1 . Therefore, it was natural for these three cases to fit the p_t spectra of π^- from midrapidity range with the one-temperature functions. As seen from Tables 7 and 10, the extracted values of the spectral temperatures T_1 and T_2 practically coincided for the target and projectile fragmentation regions. The obvious reason for this is the symmetry of the colliding ${}^{12}\text{C} + {}^{12}\text{C}$ system with identical target and projectile nuclei.

3. Summary and Conclusions

We studied the dependences of the p_t spectra of the negative pions produced in minimum bias ${}^{12}C + {}^{12}C$ collisions at $4.2A \,\text{GeV}/c$ on the collision centrality and pion rapidity range. The extracted transverse momentum spectra were analyzed using fits by Hagedorn, Boltzmann, Simple Exponential and Gaussian functions. We observed that the p_t spectra of π^- mesons are described much better by the twotemperature Hagedorn and Boltzmann functions compared to the fitting done by the one-temperature functions, which is in line with the earlier papers.^{15–17,19,20,23} Out of the four fitting functions used, Hagedorn and Boltzmann functions gave significantly better fits of the experimental p_t spectra with the physically acceptable values of the extracted spectral temperatures, compared to Simple Exponential and Gaussian functions. The fitting of the p_t spectra of π^- with Boltzmann function resulted in slightly lower values of the spectral temperatures compared to those by Hagedorn function. The fitting of the pion spectra by Gaussian function was not appropriate, since it resulted consistently in unphysically large values of T_1 and T_2 . The spectral temperatures (T_1, T_2) of π^- mesons in ${}^{12}C + {}^{12}C$ collisions at $4.2A \,\mathrm{GeV}/c$ and their relative contributions (R_1, R_2) were extracted from fitting the total p_t spectra in the whole p_t range of π^- by the two-temperature Hagedorn and the two-temperature Boltzmann functions. The dominant contribution $(R_1 \sim 85\%)$ to the total π^- multiplicity in ${}^{12}C + {}^{12}C$ collisions at $4.2A \,\text{GeV}/c$ is given by the spectral temperature $T_1 \sim (65-76) \pm 3 \,\mathrm{MeV}$, which agrees within the uncertainties with the results of Ref. 15.

We extracted and fitted the p_t spectra of the negative pions for peripheral, semicentral and central ${}^{12}\text{C} + {}^{12}\text{C}$ collisions as well as for three rapidity regions of $\pi^$ in the fitting ranges $p_t = 0.1-1.2 \text{ GeV}/c$ and $p_t = 0.1-0.7 \text{ GeV}/c$. The extracted values of the spectral temperatures T_1 and T_2 were consistently larger for the p_t spectra of π^- mesons coming from midrapidity range ($|y_{\rm cm}| \leq 0.3$) as compared to those of transverse momentum spectra of the negative pions generated in the

target $(y_{\rm cm} \leq -0.3)$ and projectile $(y_{\rm cm} \geq 0.3)$ fragmentation regions. The spectral temperatures T_1 and T_2 of the p_t spectra of π^- mesons extracted from fitting in range $p_t = 0.1-1.2 \,{\rm GeV}/c$ were very close to each other for the group of peripheral, semicentral and central ${}^{12}{\rm C} + {}^{12}{\rm C}$ collision events, selected using the number of participant protons, and thus practically did not depend on the collision centrality. However, the values of T_1 and T_2 extracted from fitting in range $p_t = 0.1-0.7 \,{\rm GeV}/c$ were consistently and noticeably larger in case of the central collisions as compared to peripheral and semicentral collisions. Possible reason for that could be that the higher p_t and high temperature part of the pion spectra with quite large statistical errors influences significantly, the extracted T_1 and T_2 values masking and suppressing the centrality dependence of the spectral temperatures. This is likely to be the reason for not observing the centrality dependence of the extracted temperatures in the earlier works, 15,23 where the pion p_t spectra for peripheral, semicentral and central collisions were fitted in the whole transverse momentum range.

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On centrality and rapidity dependences of transverse momentum spectra

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